Published by Institute of Physics Publishing for SISSA/ISAS
Received: *September 28, 2001*
Accepted: *April 11, 2002*

Duality symmetric massive type-II theories in $D = 8$ and $D = 6$*

Harvendra Singh

Fachbereich Physik, Martin-Luther-Universität Halle-Wittenberg,
Friedemann-Bach-Platz 6, D-06099 Halle, Germany
E-mail: singh@physik.uni-halle.de

Abstract: We study T^2 compactification of massive type-IIA supergravity in presence of possible Ramond-Ramond (RR) background fluxes. The resulting theory in $D = 8$ is shown to possess full $\mathrm{SL}(2, \mathbb{R}) \times \mathrm{SL}(2, \mathbb{R})$ T-duality symmetry similar to the massless case. It is shown that elements of duality symmetry interpolate between massive type IIA compactified on T^2 and ordinary type IIA compactified on T^2 with RR 2-form flux. We also discuss relationship between M-theory vacua and massive type-IIA vacua. The D8-brane is found to correspond to M-theory 'pure gravity' solution which is a direct product of 7-dimensional Minkowski space and a 4-dimensional instanton. We also construct D6-D8 bound state which preserves 1/2 supersymmetries. We then discuss massive IIA compactification on T^4 and point out that when all possible RR fluxes on T^4 are turned on the six-dimensional theory appears to assume a nice $\mathrm{SO}(4, 4)$ invariant form.

Keywords: Superstrings and Heterotic Strings, M-Theory, D-branes, String Duality.

*Work supported by: AvH – the Alexander von Humboldt foundation.

JHEP04(2002)017

Contents

1. Introduction

Recently considerable attention has been focused on the study of gauged/massive supergravity theories owing to their importance in AdS/CFT analysis [1]. In traditional methods gauged supergravity theories can be constructed out of their ungauged versions either by gauging a subgroup of the R-symmetry group and/or by gauging the isometries of the scalar manifold using the vector fields in the spectrum [2]. This procedure is incorporated in a manner such that it does not change the overall particle spectrum and the number of supersymmetries in the theory. However it does, generically, change the properties of the ground state. In most of the cases, if a minkowskian spacetime is a solution of the ungauged theory, it ceases to be a ground state in the case of gauged supergravity. Instead the supersymmetric ground states are often of the anti-de-Sitter (AdS) or domain-wall type solutions. Gauged supergravities with AdS-ground states have been focus of much attention in AdS/CFT correspondence as they can be derived through a compactification of ten or eleven dimensional supergravity theories on spacetimes involving AdS subspaces, e.g., $AdS_{p+2} \times S^{D-2-p}$ which are near horizon geometries of p-branes.

We are interested here in the study of massive supergravities which are closed relatives of gauged supergravities. In these theories some of the vector (or tensor) fields become massive upon eating other fields in their massless spectrum, analogous to a Higgs mechanism. In this procedure again the total degrees of freedom remain unaltered and so does the number of supercharges. A well studied example of a massive theory is the massive type-IIA supergravity (m-IIA) in $D = 10$ constructed by Romans [3]. In string theory massive supergravities typically can arise in lower dimensions through generalized Scherk-Schwarz reduction [4], provided that some field strength $dA_{(p)}$ of the p-form tensor field $A_{(p)}$ is given a non-trivial background value (flux) along the compact directions [5]. Such background fluxes can be turned on consistently if the action and the field equations depend on $A_{(p)}$ only through its field strength $dA_{(p)}$.

Our purpose here is to study massive supergravities in the context of string dualities and Dp-branes. The massive type-IIA supergravity has a domain wall solution which preserves 1/2 of the 32 supercharges [5] and has been given an interpretation of a type-IIA D8-brane. This observation has led to the search for possible duality connections involving massive supergravity theories analogous to the existing duality symmetries in the ordinary (massless) cases. This required the construction of new massive supergravities in lower dimensions through generalized dimensional reduction [5]–[21]. However, it has still remained an interesting open question, to what extent the generalized compactifications do respect the duality properties of the massless cases. In a parallel line of developments Calabi-Yau compactifications with background fluxes have also been studied because of their phenomenological properties [22]–[42]. One finds that background fluxes typically generate a potential for some of the moduli fields of the theory without fluxes and as a consequence the moduli space — and hence the arbitrariness of the theory — is reduced. In addition the resulting ground states can break supersymmetry spontaneously.

In this paper we study a generalized T^2 and T^4 reduction of ten-dimensional massive type-IIA supergravity with all possible R-R background fluxes turned on. Our goal is to investigate the fate of the perturbative $O(d,d)$ duality symmetries and the non-perturbative S-duality. We also discuss the relation of massive type-IIA theory with M-theory. The paper is organized as follows. In section 2 we briefly recall massive type-IIA sugra and fix our conventions to be used for the cases of toroidal reductions. Section 3 covers the compactification of massive type-IIA theory on T^2. We find that provided RR 2-form flux is turned on the resulting eight-dimensional theory can be presented in a manifestly $\mathrm{SL}(2,\mathbb{R})\times \mathrm{SL}(2,\mathbb{R})$ invariant form. In section 4 we study the vacuum solutions of this massive 8-dimensional supergravity and relate them to the solutions of ordinary IIA by using the elements of T-duality group. We uplift these ordinary IIA vacua to eleven dimensions. Particularly the D8-brane is shown to correspond to a pure gravity solution in eleven dimensions which contains an domain-wall-type instanton line element. Thus perturbative T-duality symmetry interpolates between vacua of massive and massless type-IIA theories on T^2. This property also allows us to further relate massive type-II vacua to eleven dimensional solutions. In section 5 we obtain a D6-D8-brane bound state which preserves 16 supercharges. In section 6 we outline the compactification on T^4 and point out that the massive IIA theory on T^4 will have a full $\mathrm{SO}(4,4)$ symmetry provided all the RR fluxes for all the fields are turned on. Finally we have summarized our results in the section 7.

2. Review

The type-IIA supergravity in ten dimensions, which describes the low energy limit of type-IIA superstrings, contains in the massless bosonic spectrum the graviton $\hat{g}_{MN}$, the dilaton $\hat{\phi}$, NS-NS two-form $\hat{B}_{(2)}$, a R-R one-form $\hat{A}_{(1)}$ and a R-R three-form $\hat{C}_{(3)}$. The fermionic fields consist of two gravitini and two Majorana $\frac{1}{2}$-spinors. Massive type-IIA supergravity (m-IIA) [3] is a generalization of that to include a mass term for the $\hat{B}$-field without disturbing the supersymmetry. More precisely, the $\hat{B}$-field becomes massive through a Higgs type mechanism in which it eats the vector field $\hat{A}$. The supersymmetric bosonic action for massive IIA theory in the string frame can be written as[1]

$$
\begin{aligned}
S = \int \Big[& e^{-2\hat{\phi}} \left\{ \frac{1}{4}\hat{R}\ \hat{*}1 + d\hat{\phi}\ \hat{*}d\hat{\phi} - \frac{1}{2}\hat{H}_{(3)}\ \hat{*}\hat{H}_{(3)} \right\} - \frac{1}{2}\hat{F}_{(2)}\ \hat{*}\hat{F}_{(2)} - \frac{1}{2}\hat{F}_{(4)}\ \hat{*}\hat{F}_{(4)} - \frac{m^2}{2}\ \hat{*}1 + \\
& + d\hat{C}_{(3)} d\hat{C}_{(3)} \hat{B}_{(2)} + 2 d\hat{C}_{(3)} d\hat{A}_{(1)} \hat{B}^2_{(2)} + \frac{4}{3} d\hat{A}_{(1)} d\hat{A}_{(1)} \hat{B}^3_{(2)} + \frac{4}{3} m d\hat{C}_{(3)} \hat{B}^3_{(2)} + \\
& + 2 m d\hat{A}_{(1)} \hat{B}^4_{(2)} + \frac{4}{5} m^2 \hat{B}^5_{(2)} \Big] ,
\end{aligned}
\tag{2.1}
$$

where m is the mass parameter. The various field strengths in the action (2.1) are given by

$$
\hat{H}_{(3)} = d\hat{B}_{(2)} , \qquad \hat{F}_{(2)} = d\hat{A}_{(1)} + 2m\hat{B}_{(2)} , \qquad \hat{F}_{(4)} = d\hat{C}_{(3)} + 2\hat{B}_{(2)} d\hat{A}_{(1)} + 2m\hat{B}^2_{(2)} . \tag{2.2}
$$

Note that potentials $\hat{A}$ and $\hat{C}$ appear only through their derivatives in the action (2.1) and thus obey the standard p-form gauge invariance $A_{(p)} \to A_{(p)} + d\lambda_{(p-1)}$. The two-form $\hat{B}$ on the other hand also appears without derivatives but nevertheless the 'Stueckelberg' gauge transformation

$$
\delta\hat{A} = -2m\lambda_{(1)} , \qquad \delta\hat{B} = d\lambda_{(1)} , \qquad \delta\hat{C} = -2\lambda_{(1)} d\hat{A} \tag{2.3}
$$

leaves the action invariant.

As we shall turn to the compactification in the later sections let us recall here some facts about toroidal compactifications. Standard Kaluza-Klein reduction considers the theory in a spacetime background $M_D \times T^d$, where M_D is a non-compact D-dimensional manifold with lorentzian signature while T^d is a d-dimensional compact torus. This ansatz is consistent whenever the spacetime background satisfies the $(D+d)$-dimensional field equations. However, for massive type IIA there are no direct product $M_D \times T^d$ solutions; instead the ground states are domain-wall (D8-brane) solutions [5], see section 4. It has been discussed in [5] that Kaluza-Klein reduction can still be carried out for such theories. The T^1 compactification of massive type IIA has been discussed at great length in [5]. These ideas have also been applied to K3 compactifications recently [18].

Thus for the 10-dimensional zehnbein we take the standard toroidal ansatz

$$
\hat{e}_M^{\hat{a}}\,(x,y) = \begin{pmatrix} e_\mu^a(x) & e_m^i K_\mu^m \\ 0 & e_m^i \end{pmatrix} , \tag{2.4}
$$

where coordinates y^m $(m = 1, \ldots, d)$ are tangent to the tori. The internal metric on tori is given by $G_{mn}(x) = e_m^i \delta_{ij} e_n^j$ while the D-dimensional Minkowski metric is $g_{\mu\nu}(x) = e_\mu^a \eta_{ab} e_\nu^b$. K_μ^m are the Kaluza-Klein gauge fields. Let us define gauge invariant d one-forms $\eta_{(1)}^m = dy^m + K_{(1)}^m$.

[1] Our conventions are same as in [18] where every product of forms is understood to be a wedge product. We denote a p-form with a lower index like (p) and later drop it, for simplification, when it becomes obvious.

JHEP04(2002)017

The standard toroidal ansatz for the dilaton and the NS-NS two-form $\hat{B}$ is

$$\hat{\phi}(x,y) = \hat{\phi}(x)\,, \qquad \hat{B}_{(2)}(x,y) = B_{(2)}(x) + \bar{A}_m(x)\;\eta^m + \frac{1}{2}B_{mn}(x)\;\eta^m\eta^n\,, \tag{2.5}$$

where $B_{(2)} = \bar{B}_{(2)} - \frac{1}{2}\bar{A}_m K^m$ with $\bar{B}$ being a 2-form in D dimensions, $\bar{A}_m$ are d vector potentials and the B_{mn} represents scalar fields antisymmetric in m,n indices ($m,n = 1,\ldots,d$). With these ansätze the NS-NS part of the action (2.1) reduces as follows [43]

$$\int e^{-2\phi}\Big[\frac{1}{4}R\;{}^*1 + d\phi\;{}^*d\phi - \frac{1}{2}H_3\;{}^*H_3 + \frac{1}{16}dG^{mn}\;{}^*dG_{mn} - \frac{1}{4}dB_{mn}\;{}^*dB_{pq}G^{mp}G^{nq} - \\ - \frac{1}{8}dK^m\;{}^*dK^nG_{mn}\frac{1}{2}(d\bar{A}_m - B_{mp}dK^p)\;{}^* - (d\bar{A}_n - B_{nq}dK^q)G^{mn}\Big]\,, \tag{2.6}$$

with

$$2\phi = 2\hat{\phi} - \frac{1}{2}\ln G\,, \qquad H_{(3)} = d\bar{B} - \frac{1}{2}(\bar{A}_m dK^m + K^m d\bar{A}_m)\,, \tag{2.7}$$

where G represents the determinant of the internal metric.

In the next sections we shall consider the specific cases where we consider the compactifications of m-IIA on even tori. We are focusing on even tori as we are interested in generalized reduction with the presence of RR-fluxes. Note that type IIA involves only even-form field strengths in the RR sector. The compactification of m-IIA on T^1 has been discussed in [5] which also involves a generalized reduction of type IIB in order to study duality symmetry. Similarly a T^3 reduction of m-IIA has to deal with the generalized reduction of type IIB on T^3.

3. Compactification on T^2

Here we specifically consider the case of compactification on a 2-torus. For the one-form $\hat{A}_{(1)}$ and the three-form $\hat{C}_{(3)}$ we would consider a generalized Kaluza-Klein ansatz where background fluxes are considered. This generalization is possible since $\hat{A}_{(1)}$ and $\hat{C}_{(3)}$ appear in the action (2.1) only through derivatives, therefore an appropriate background value can be consistently turned on. We take the following generalized ansatz

$$\begin{aligned}\hat{A}(x,y) &= A_{(1)}(x) + A_{(0)m}(x,y)dy^m\,,\\ \hat{C}(x,y) &= C_{(3)}(x) + T_{(2)m}(x)dy^m + \frac{1}{2}V_{(1)mn}(x)dy^m dy^n\,.\end{aligned} \tag{3.1}$$

Note that the scalar-forms are allowed to retain a dependence on the coordinates of the torus unlike in standard toroidal reduction. The consistency of toroidal reduction requires it to be at most a linear dependence on the torus coordinates, we fix it to be

$$A_m(x,y) = a_m(x) - \frac{1}{2}m_{mn}y^n\,, \tag{3.2}$$

where constants m_{mn} are antisymmetric in indices ($m,n = 1,2$). This will give us

$$\begin{aligned}\hat{d}\hat{A} &= \mathcal{D}A + \mathcal{D}a_m\eta^m + \frac{1}{2}m_{mn}\eta^m\eta^n\,,\\ \hat{d}\hat{C} &= \mathcal{D}C_{(3)}(x) + \mathcal{D}T_{(2)m}\eta^m + \frac{1}{2}dV_{(1)mn}\eta^m\eta^n\,,\end{aligned} \tag{3.3}$$

where various $\mathcal{D}$-derivatives are defined as

$$\begin{aligned}
\mathcal{D}A &= dA - da_m K^m + \frac{1}{2} m_{mn} K^m K^n\,, & \mathcal{D}a_m &= da_m + m_{mn} K^n\,, \\
\mathcal{D}T_m &= dT_m + dV_{mn} K^n\,, & \mathcal{D}C &= dC - dT_m K^m + \frac{1}{2} dV_{mn} K^m K^n\,.
\end{aligned} \tag{3.4}$$

Note that various forms can be distinguished by their symbols and the internal indices they carry. Thus we see explicitely that even if the potential $\hat{A}$ in (3.1) depends upon the torus coordinates, the derivatives in (3.3) do not. This dependence also drops out in the action and the field equations. Through above generalized ansatz we have effectively introduced one new parameter m_{12} in the guise of 2-form flux. This generalization has been possible only because $\hat{A}_1$ appears in the action covered with derivative and T^2 has one 2-cycle along which an appropriate background flux could be turned on.

The massive field strengths in (2.2) become

$$\begin{aligned}
\hat{F}_{(2)} &= (\mathcal{D}A + 2mB) + (\mathcal{D}a_n + 2m\bar{A}_m)\eta^m + \frac{1}{2}(m_{mn} + 2mB_{mn})\eta^m\eta^n\,, \\
\hat{F}_{(4)} &= (\mathcal{D}C + 2B\mathcal{D}A + 2mBB) + (\mathcal{D}T_n + 2B\mathcal{D}a_m + 2\bar{A}_m\mathcal{D}A + 4mB\bar{A}_m)\eta^m + \\
&\quad + \frac{1}{2}(\mathcal{D}V_{mn} - 4\bar{A}_m\mathcal{D}a_n + 2B_{mn}\mathcal{D}A + 2Bm_{mn} + 4mBB_{mn} - 4m\bar{A}_m\bar{A}_n)\eta^m\eta^n\,.
\end{aligned} \tag{3.5}$$

Altogether, the bosonic spectrum of the reduced 8-dimensional theory consists of the graviton $g_{\mu\nu}$, dilaton ϕ, 2-form $\bar{B}$, 4 scalars and 4 vectors $(\bar{A}_m, K^m)$ in the NS-NS sector. From the R-R sector we have 2 scalars a_m, 2 vectors (V_{12}, A), 2 tensors T_m and one 3-form C whose field strength is (anti)self-dual in $D = 8$. Also we have two parameters m_{12} and m. This is the bosonic content of maximal type-II supergravity theory in $D = 8$. In the massless case these fields fit into various representation of the T-duality group $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R}) \sim \mathrm{SO}(2,2)$. Here too various fields combine into the $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$ representations as

$$\begin{aligned}
A^{ru}_{(1)} &= (A^{r1}, A^{r2})\,, \quad A^{ru=1} = (\bar{A}_1, \bar{A}_2)\,, \quad A^{ru=2} = \left(\frac{K^2}{2}, -\frac{K^1}{2}\right), \\
a^r_{(0)} &= (a_1, a_2)\,, \quad t^r_{(2)} = (T_1, T_2)\,, \quad \mathcal{A}^u_{(1)} = (-V_{12}, A)\,, \quad m^u = (-m_{12}, m)\,,
\end{aligned} \tag{3.6}$$

where indices $r = 1, 2$ belong to first $\mathrm{SL}(2,\mathbb{R})$ while indices $u = 1, 2$ belong to the second $\mathrm{SL}(2,\mathbb{R})$ group. Note that the mass and flux parameters also fit into a fundamental representation of $\mathrm{SL}(2,\mathbb{R})$.

In order to obtain the action of the massless modes for this theory we substitute the ansatz (2.4)-(3.3) into the action (2.1). The resulting eight-dimensional bosonic action reads in the kinetic part

$$\begin{aligned}
S_{D=8} = \int \Bigg[&\frac{1}{4} e^{-2\phi} \Big\{ R\, {}^*1 + 4\, d\phi\, {}^*d\phi - 2H_3\, {}^*H_3 - 2dA^{ru}\, {}^*dA^{sv} M^{-1}_{rs} \mathcal{M}^{-1}_{uv} + \\
&+ \frac{1}{4} \mathrm{Tr}\, d\mathcal{M}^{-1}\, {}^*d\mathcal{M} + \frac{1}{4} \mathrm{Tr}\, dM^{-1}\, {}^*dM) \Big\} - \frac{1}{2} \mathcal{G}\, {}^*\mathcal{G} \sqrt{G} - \\
&- \frac{1}{2} \mathcal{F}^r_{(3)} (M^{-1})_{rs}\, {}^*\mathcal{F}^s_{(3)} - \frac{1}{2} \mathcal{F}^u_{(2)} (\mathcal{M}^{-1})_{uv}\, {}^*\mathcal{F}^v_{(2)} - \\
&- \frac{1}{2} \mathcal{F}^r_{(1)} (M^{-1})_{rs}\, {}^*\mathcal{F}^s_{(1)} - \frac{1}{2} m^u (\mathcal{M}^{-1})_{uv}\, {}^*m^v \Bigg] + S_{C-S}\,,
\end{aligned} \tag{3.7}$$

where the Chern-Simon part of the action is

$$
\begin{aligned}
S_{C-S} &= \\
&= \int -b\mathcal{G}\mathcal{G} + \mathcal{G}\left\{-2(\mathcal{D}T_m + 2\mathcal{D}a_m B + \mathcal{D}A\bar{A}_m + 2mB\bar{A}_m)\bar{A}_n + dV_{mn}B + m_{mn}BB\right\}\epsilon^{mn} + \\
&\quad + \Bigg[-\mathcal{D}T_m\mathcal{D}T_n B - \{2\mathcal{D}T_m\mathcal{D}a_n + \mathcal{D}V_{mn}\mathcal{D}A - 2\mathcal{D}A\mathcal{D}AB_{mn}\}(B)^2 + \\
&\qquad + \left\{4m\mathcal{D}AB_{mn} - \frac{4}{3}m\mathcal{D}V_{mn} - \frac{2}{3}m_{mn}\mathcal{D}A - \frac{4}{3}\mathcal{D}a_m\mathcal{D}a_n\right\}(B)^3 + \\
&\qquad + (2m^2 B_{mn} - m m_{mn})(B)^4\Bigg]\epsilon^{mn}\,,
\end{aligned}
\tag{3.8}
$$

with $\epsilon^{12} = 1$ and we have defined $b \equiv -B_{12}$,

$$
2\phi = 2\hat{\phi} - \frac{1}{2}\ln G\,, \qquad H_{(3)} = d\bar{B} + A^{ru}dA^{sv}L_{rs}\mathcal{L}_{uv}\,.
$$

Various R-R field strengths in the above action are

$$
\begin{aligned}
\mathcal{F}^r_{(1)} &= da^r + 2m^u\ A^r_u\,, \\
\mathcal{F}^u_{(2)} &= d\mathcal{A}^u - 2da^r A^u_r - 2m^v A^r_v A^u_r + 2m^u\bar{B}\,, \\
\mathcal{F}^r_{(3)} &= dt^r + 2d\mathcal{A}^u A^r_u - 2da^s A^u_s A^r_u - \frac{4}{3}m^v A^s_v A^u_s A^r_u + 2\bar{B}\mathcal{F}^r_{(1)}\,, \\
\mathcal{G}_{(4)} &= \mathcal{D}C + 2B\mathcal{D}A + 2mBB\,.
\end{aligned}
\tag{3.9}
$$

The indices r and u can be raised or lowered by the use of two $\mathrm{SL}(2,\mathbb{R})$ metrics L and $\mathcal{L}$, respectively. The two metrics are given by

$$
L_{rs} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \equiv \mathcal{L}_{uv}\,.
$$

The uni-modular matrices which belong to two $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{SO}(2)$ cosets are given by

$$
M^{-1} = \sqrt{G}\begin{pmatrix} G^{11} & G^{12} \\ G^{21} & G^{22} \end{pmatrix}, \qquad \mathcal{M}^{-1} = \frac{1}{\sqrt{G}}\begin{pmatrix} 1 & 2b \\ 2b & 4(b)^2 + G \end{pmatrix}, \tag{3.10}
$$

and they satisfy $M^T L M = L$, $\mathcal{M}^T\mathcal{L}\mathcal{M} = \mathcal{L}$.

Under an $\mathrm{SL}(2,\mathbb{R})$ transformation

$$
\begin{aligned}
&\mathcal{M} \to \Lambda\mathcal{M}\Lambda^T\,, \qquad \mathcal{A}^u \to \Lambda^u{}_v\mathcal{A}^v\,, \qquad A^{ru} \to \Lambda^u{}_v A^{rv}\,, \\
&m^u \to \Lambda^u{}_v m^v\,,
\end{aligned}
\tag{3.11}
$$

with $\Lambda\mathcal{L}\Lambda^T = \mathcal{L}$. The same is true for other $\mathrm{SL}(2,\mathbb{R})$ group with acts upon r, s indices.

Note that the kinetic terms in the action except the terms involving 4-form field strength $\mathcal{G}$ remain invariant under the action of above T-duality group. We have not provided the explicit invariant form for the Chern-Simon terms which also remain invariant provided the flux m_{12} and mass m transform as an $\mathrm{SL}(2,\mathbb{R})$ doublet. It remains to be seen if the field equations and the Bianchi identity for 3-form potential C transform covariantly. Let us write down the field equation for C. From the 8-dimensional action

JHEP04(2002)017

in (3.7) we get

$$d\Big[-(\sqrt{G}\ {}^*\mathcal{G}+2b\mathcal{G})+\Big[-2(\mathcal{D}T_m+2\mathcal{D}a_mB+\mathcal{D}A\bar{A}_m+2mB\bar{A}_m)\bar{A}_n+ \\ +dV_{mn}B+m_{mn}BB\Big]\epsilon^{mn}\Big]=0\equiv d(d\tilde{C}_{(3)})\,, \tag{3.12}$$

where $\tilde{C}_{(3)}$ is defined to be the dual 3-form potential. If we now define $\mathcal{G}^1=-\sqrt{G}\ {}^*\mathcal{G}-2b\mathcal{G}$ and $\mathcal{G}^2=\mathcal{G}$, then field equation (3.12) simply becomes a Bianchi identity for $\tilde{C}_{(3)}$. This Bianchi for $\tilde{C}_{(3)}$ and the Bianchi identity for $C_{(3)}$, which can be derived from its field strength in (3.9), form an SL$(2,\mathbb{R})$ covariant set of equations. From this we can write down SL$(2,\mathbb{R})$ 4-form field strength as

$$\begin{aligned}\mathcal{G}^u &= d\mathcal{C}^u-2dt_rA^{ru}-2d\mathcal{A}_vA^v_rA^{ru}+\frac{4}{3}da_rA^r_vA^v_sA^{su}+ \\ &+\frac{2}{3}m_wA^w_rA^r_vA^v_sA^{su}+2\mathcal{F}^u_{(2)}\bar{B}+2m^u(\bar{B})^2\,,\end{aligned} \tag{3.13}$$

where 3-forms are $\mathcal{C}^u=(\tilde{C},C)$.

The action (3.7) will possess Stueckelberg gauge invariances which is obvious from the investigation of the field strengths in eq. (3.9). Through these gauge invariaces, the vector fields A^{ru} can eat the scalars a^r and can become massive. Similarly the tensor field $\bar{B}$ can eat one of the vector fields $\mathcal{A}^u$ and can become massive. Explicit gauge transformations of the fields can be derived from those in (2.3).

This completes our analysis of the eight-dimensional massive type-II supergravity action which we have shown to possess an explicit SL$(2,\mathbb{R})\times$ SL$(2,\mathbb{R})$ T-duality symmetry provided the flux m_{12} and the m-IIA mass m transform as SL$(2,\mathbb{R})$ doublet. Note that this restoration of the T-duality symmetry of the massive II theory in $D=8$ is direct consequence of our ansatz in (3.2). Under this structure the mass and the flux behave in identical manner. If we start without any flux we can generate a flux by making an SL$(2,\mathbb{R})$ rotation, see next section. In other words, if we compactify m-IIA on T^2 without RR flux, the resulting massive theory in $D=8$ can be mapped to ordinary IIA theory compactified on T^2 with RR 2-form flux, by making use of the duality symmetry.

We are not surprised with this identification between flux and mass. It is the repetition of the story when m-IIA is compactified on a circle of radius R and type-IIB strings compactified on a circle of radius $1/R$ along with RR 1-form (axionic scalar) flux. This identification has been crucial in order to make massive T-duality in $D=9$ manifest [5]. Similar phenomenon has been shown to be repeated for the case of compactification on $K3$ manifold also [18].

4. D8-brane vs M-theory instanton

The ten-dimensional massive IIA supergravity theory has D8-brane (domain-wall) solutions which preserve sixteen supercharges [5]. In the string frame metric the solution is given by

$$\begin{aligned}d\hat{s}^2 &= H^{-1/2}(-dt^2+dx_1^2+\cdots+dx_6^2+dy_1^2+dy_2^2)+H^{1/2}dz^2\,, \\ 2\hat{\phi} &= -\frac{5}{2}\ln H\,,\end{aligned} \tag{4.1}$$

where $H = \text{const.} + 2m|z - z_0|$ is a harmonic function of only the transverse coordinate z and all other fields have vanishing background values, z_0 refers to the location of the domain-wall. We compactify this solution by wrapping two of its world-volume directions, say y_1, y_2, on T^2. The corresponding 8-dimensional domain-wall (or 6-brane) solution can be written down using our analysis of the last section

$$\begin{aligned} ds_8^2 &= H^{-1/2}\left(-dt^2 + \sum_{i=1}^{6} dx_i^2\right) + H^{1/2}dz^2\,, \\ 2\phi &= 2\hat{\phi} - \frac{1}{2}\ln G = -2\ln H\,, \\ m^u &= (0, m)\,, \qquad \mathcal{M} = \mathrm{diag}(H^{-\frac{1}{2}}, H^{\frac{1}{2}})\,, \end{aligned} \tag{4.2}$$

while other 8-dimensional background fields vanish. Clearly this vacuum configuration corresponds to the situation when there is no background flux. The solution (4.2) still has 16 unbroken supersymmetries. Let us note that similar domain-wall type solutions appear in various situations and in one such case domain-wall solution are discussed in [6] where a massive 8-dimensional theory is obtained from generalized reduction from 9-dimensional type-II theory to eight dimensions.

Now, by applying an $\mathrm{SL}(2,\mathbb{R})$ transformation (3.11) on the background fields in (4.2) new solutions with non-trivial R-R flux can be generated. Let us consider the special case of $\mathrm{SL}(2,\mathbb{R})$ transformation given by

$$\Lambda = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{4.3}$$

Inserting Λ and the configuration (4.2) in (3.11) we get

$$m^u = \begin{pmatrix} 0 \\ m \end{pmatrix} \to m'^u = \begin{pmatrix} -m \\ 0 \end{pmatrix}, \qquad \mathcal{M} = \begin{pmatrix} H^{-\frac{1}{2}} & 0 \\ 0 & H^{\frac{1}{2}} \end{pmatrix} \to \mathcal{M}' = \begin{pmatrix} H^{\frac{1}{2}} & 0 \\ 0 & H^{-\frac{1}{2}} \end{pmatrix}, \tag{4.4}$$

while eight-dimensional metric and the dilaton remain invariant. The transformed mass vector m'^u implies that the new configuration is a solution of a massless IIA compactified on T^2 with 2-form flux along T^2.

Now lifting the solution (4.4) back to ten dimensions, we get the following type-IIA configuration (we write them with a prime)

$$\begin{aligned} d\hat{s}'^2 &= H^{-1/2}\left(-dt^2 + \sum_{i=1}^{6} dx_i^2\right) + H^{1/2}(dz^2 + dy_1^2 + dy_2^2)\,, \\ 2\hat{\phi}' &= -\frac{3}{2}\ln H\,, \qquad \hat{F}'_{(2)} = m dy_1 \wedge dy_2\,. \end{aligned} \tag{4.5}$$

According to our ansatz in (3.2) it corresponds to $\hat{A}'_{(1)} = m y_{[1} dy_{2]}$. Since under the $\mathrm{SL}(2,\mathbb{R})$ transformation (4.4), $G \to 1/G$, it amounts to making T-duality along both the torus directions, therefore the background (4.5) represents an stack of D6-branes in ten dimensions filling the transverse T^2 and have non-trivial 2-form flux.[2] Since this solution is obtained by incorporating T-duality transformation (3.11) the number of preserved supercharges remains unchanged. Soon we will show that the type-IIA solution in (4.5) becomes an instanton in M-theory set-up.

Thus by making an $\mathrm{SL}(2,\mathbb{R})$ transformation we have transformed 8-brane solution of m-IIA into a 6-brane solution of ordinary type IIA which is supported by a non-trivial 2-form flux. Thus, the 8-dimensional perturbative duality interpolates between vacua of massive type IIA and ordinary type IIA. It is similar to the situation encountered in the case of massive type-II duality in $D = 9$ [5]. Further solutions in $D = 8$ and $D = 10$ can be generated by using other elements of the duality group which mix the mass m with the flux m_{12}. We shall consider one example of this type in the next section.

M-theory instanton. Since ordinary type-IIA theory compactified over T^2 is equivalent to M-theory on $S^1 \times T^2$, the solutions of massive IIA can be lifted to eleven dimensions by first mapping them to the solutions of ordinary IIA by using the $\mathrm{SL}(2,\mathbb{R})$ element (4.3) and then lifting them to eleven dimensions. The configuration in eq. (4.5) is a massless IIA background and can be lifted to eleven dimensions. Correspondingly we get the following eleven dimensional solution

$$\begin{aligned} ds_{11}^2 &= e^{\frac{4\hat{\phi}}{3}}(dx_{11} + 2\hat{A}_M dx^M)^2 + e^{-\frac{2\hat{\phi}}{3}} ds_{10}^2 \\ &= H^{-1}\left(dx_{11} + m(y_1 dy_2 - y_2 dy_1)\right)^2 + H(dz^2 + dy_1^2 + dy_2^2) - dt^2 + \sum_{i=1}^{6} dx_i^2 \,, \\ H &= \text{const.} + 2m|z - z_0| \,, \end{aligned} \tag{4.6}$$

where x_{11} is the coordinate of 11-dimensional circle S^1, y_1 and y_2 are also periodic while other fields vanish. This is a pure gravity solution in eleven dimensions and has the geometry which is a product of a 4-dimensional euclidean instanton (we clarify next why we call it an instanton), $\mathcal{E}^4$, and a 7-dimensional Minkowski space, M_7.

Let us focus on the properties of the instanton line element in the above, we rewrite it as

$$\begin{aligned} ds_{\text{Instanton}}^2 &= H^{-1}(d\tau + K)^2 + H(dz^2 + dy_1^2 + dy_2^2) \,, \\ H &= \text{const.} + 2m|z - z_0| \,, \qquad K = m(y_1 dy_2 - y_2 dy_1) \end{aligned} \tag{4.7}$$

with $\tau = x_{11}$ being periodic. At closer examination we find that this (y_1, y_2 are compact coordinates but their radii can be taken large enough) looks like a domain-wall-type generalization of Taub-NUT instantons [44]

$$ds_{T-N}^2 = V^{-1}(d\tau + \Omega(x))^2 + V(dx_1^2 + dx_2^2 + dx_3^2) \,, \tag{4.8}$$

in which $V(x) = \epsilon + \frac{2m}{|x - x_0|}$, with V and the 1-form Ω satisfying $dV = {}^*d\Omega$. The Hodge-dual is defined over flat transverse x-space. Precisely in the same way, H and K in (4.7) also

[2] Note, there is not much distinction between these domain-wall-type D6-branes and the usual (asymptotically flat) D6-branes which depend on all the transverse coordinates and are magnetic duals of D0-branes. The domain-wall-type D6-branes are equally possible in type-IIA theory and are BPS objects.

satisfy an identical relation

$$dH = {}^*dK\,,$$

where Hodge-dual is defined over flat z, y_1, y_2 space. Moreover the spin connections and the curvature 2-forms of (4.7) are self-dual

$$\begin{aligned}
&\omega^1{}_2 = \omega^0{}_3 = -\frac{m}{H^{3/2}}e^0\,, \qquad \omega^3{}_1 = \omega^0{}_2 = -\frac{m}{H^{3/2}}e^1\,, \qquad \omega^2{}_3 = \omega^0{}_1 = \frac{m}{H^{3/2}}e^2\,,\\
&R^1{}_2 = R^0{}_3 = \frac{4m^2}{H^3}(e^3e^0 + e^2e^1)\,, \qquad R^3{}_1 = R^0{}_2 = \frac{2m^2}{H^3}(e^3e^1 + e^0e^2)\,,\\
&R^2{}_3 = R^0{}_1 = \frac{2m^2}{H^3}(e^2e^3 + e^0e^1)\,,
\end{aligned} \tag{4.9}$$

with the basis $e^0 = H^{-\frac{1}{2}}(d\tau + my_1dy_2 - my_2dy_1)$, $e^1 = H^{\frac{1}{2}}dy_1$, $e^2 = H^{\frac{1}{2}}dy_2$, $e^3 = H^{\frac{1}{2}}dz$. Thus the line element in (4.7) is essentially a Taub-NUT instanton in four dimensions, and is of a new kind in that it has a domain-wall type extent. Thus we call it a *domain-wall-instanton*.

We note that a 11-dimensional solution almost similar to (4.6) also appear in [6, 8], perhaps authors miss in identifying them as instanton line elements. The line element in (4.6) differs in the structure of the connection 1-form K from the previous occasions. The present form of K crucially depends on our reduction ansatz in (3.2). This affects the periodicity of the coordinate, τ, of the circle S^1 which is fibred over the base z, y_1, y_2. We note that the periodicity of coordinate τ is independent of the periodicity of the T^2 coordinates y_1 and y_2 which is not the case discussed in [8]. That is we can choose the periodicity of T^2 independent of S^1. It is essential in order to make the connection between m-IIA and M-theory. The duality element (4.3) takes T^2 to its inverse size when we relate D8-brane with D6-brane which is subsequently oxidised to M-theory. Thus to get a decompactified D8-brane of m-IIA, T^2 on the M-theory side must be taken to zero size independently of the size of S^1 (which also goes to zero in type-IIA limit), and the vice-versa. This zero area limit has also been emphasized in the work of Chris Hull [15] although from the perspective of F-theory, as the analysis there deals with generalized compactification of IIB on S^1 [5]. While in present set-up we do not encounter that situation as we all the time remain in IIA set-up.

Thus D8-branes of massive type-IIA theory emerge from purely geometrical considerations of the M-theory such that it involves ‘domain-wall-instanton’. Thus we are tempted to claim that M-theory compactifications on $M_7 \times \mathcal{E}_4$ will correspond to massive IIA compactification on 2-torus. The sketch below is an effort to summarize the whole picture

$$\begin{array}{ccc}
 & & \boxed{\text{M-theory on } M_7 \times \mathcal{E}_4}\\
 & & \updownarrow\\
\boxed{\text{massive IIA on } T^2} & \xleftrightarrow{(G\leftrightarrow 1/G)} & \boxed{\text{IIA on } T^2 \text{ \& 2-form-Flux}}
\end{array}$$

5. D6-D8 bound state

In the above we have studied the background configurations which either have mass or flux but not the both. By making a more general $\mathrm{SL}(2,\mathbb{R})$ transformation we can generate vacuas in which both mass and the flux are nontrivial. Let us make the following $SL(2,\mathbb{R})$ rotaion

$$\Lambda = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}. \tag{5.1}$$

Using the transformations (3.11) and applying them on the configuration in (4.2), we get the transformed configuration with

$$m^u = \begin{pmatrix} m\sin\theta \\ m\cos\theta \end{pmatrix}, \qquad \mathcal{M} = \begin{pmatrix} H^{-\frac{1}{2}}\cos^2\theta + H^{\frac{1}{2}}\sin^2\theta & (-H^{-\frac{1}{2}} + H^{\frac{1}{2}})\cos\theta\sin\theta \\ (-H^{\frac{-1}{2}} + H^{\frac{1}{2}})\cos\theta\sin\theta & H^{-\frac{1}{2}}\sin^2\theta + H^{\frac{1}{2}}\cos^2\theta \end{pmatrix}, \tag{5.2}$$

while the eight-dimensional metric and dilaton remain same as in (4.2). Uplifting (5.2) to ten dimensions gives us

$$\begin{aligned} d\hat{s}^2_{10} &= H^{1/2}\left\{H^{-1}(-dt^2 + dx_i^2) + H'^{-1}(dy_1^2 + dy_2^2) + dz^2\right\}, \\ e^{2\hat{\phi}} &= g_s^2 H^{-3/2} H'^{-1}, \qquad d\hat{A}_{(1)} = -\frac{1}{g_s} m\sin\theta dy_1 \wedge dy_2, \\ 2\hat{B}_{y_1y_2} &= \tan\theta\left(1 + \frac{1}{H-1}\frac{1}{\cos^2\theta}\right)^{-1}, \end{aligned} \tag{5.3}$$

where $H = \text{const.} + 2m|z - z_0|$ and $H' = \cos^2\theta(H-1) + 1$. We have introduced the parameter g_s which represents string coupling constant. Since this configuration is a solution of massive IIA theory and also has nontrivial 2-form flux, our interpretation is that (5.3) represents a bound state of D6 and D8 branes. Moreover it preserves 16 supercharges. Note that the asymptotic ($z \to \infty$) value of $\hat{B}$ is proportional to $tan\theta$ which is usually the case with D$(p-2)$-Dp bound states for $(2 \le p \le 6)$ [45]. It would be therefore interesting to study the non-commutative Yang-Mills (NCYM) decoupling limit [46] for the D6-D8 bound state in (5.3), which we leave for a later investigation [47].

6. Compactification on T^4

Through a similar procedure a generalized compactification of m-IIA theory with R-R fluxes can be carried out on a four-torus as well. On a four-torus we have six 2-cycles and one 4-cycle. So we can switch on fluxes correspoding to each of these cycles. This will add seven new flux parameters in the six-dimensional theory. These seven parameters and the mass will combine into an eight-dimensional spinorial representation of the $SO(4,4)$ T-duality group. A generalized Kaluza-Klein ansatz can be written as

$$\begin{aligned} \hat{A}(x,y) &= A(x) + (a_m(x) - \frac{1}{2}m_{mn}y^n)dy^m, \\ \hat{C}(x,y) &= C_{(3)}(x) + T_{(2)m}(x)dy^m + \frac{1}{2}V_{(1)mn}(x)dy^m dy^n + \\ &\quad + \frac{1}{3!}\left(s_{(0)mnp}(x) - \frac{1}{4}\beta_{mnpq}y^q\right)dy^m dy^n dy^p, \end{aligned} \tag{6.1}$$

where all the scalar-forms are allowed to retain up to a linear dependence on the coordinates of the torus. Flux parameters m_{mn} and β_{mnpq} are completely antisymmetric in their indices, $m, n = 1, 2, 3, 4$. Then

$$
\begin{aligned}
\hat{d}\hat{A} &= \mathcal{D}A + \mathcal{D}a_m \eta^m + \frac{1}{2} m_{mn} \eta^m \eta^n , \\
\hat{d}\hat{C} &= \mathcal{D}C_{(3)}(x) + \mathcal{D}T_{(2)m}\eta^m + \frac{1}{2}\mathcal{D}V_{(1)mn}\eta^m\eta^n + \\
&\quad + \frac{1}{3!}\mathcal{D}s_{(0)mnp}\eta^m\eta^n\eta^p + \frac{1}{4!}\beta_{mnpq}\eta^m\eta^n\eta^p\eta^q
\end{aligned}
\tag{6.2}
$$

where various derivatives $\mathcal{D}$ are given by

$$
\begin{aligned}
\mathcal{D}A &= dA - da_m K^m + \frac{1}{2} m_{mn} K^m K^n , \qquad \mathcal{D}a_m = da_m + m_{mn} K^n , \\
\mathcal{D}s_{mnp} &= ds_{mnp} + \beta_{mnpq} K^q \\
\mathcal{D}V_{mn} &= dV_{mn} - ds_{mnp} K^p + \frac{1}{2}\beta_{mnpq} K^p K^q , \\
\mathcal{D}T_m &= dT_m + dV_{mn} K^n + \frac{1}{2} ds_{mnp} K^n K^p + \frac{1}{3!}\beta_{mnpq} K^n K^p K^q , \\
\mathcal{D}C &= dC - dT_m K^m + \frac{1}{2!} dV_{mn} K^m K^n - \frac{1}{3!} ds_{mnp} K^m K^n K^p + \frac{1}{4!}\beta_{mnpq} K^m K^n K^p K^q .
\end{aligned}
\tag{6.3}
$$

Thus we see explicitely that even if the potential depends upon the compact torus coordinates but this dependence is dropped out in their derivatives so also in the field equations. Correspondingly, massive field strengths in (2.2) become

$$
\begin{aligned}
\hat{F}_{(2)} &= (\mathcal{D}A + 2mB) + (\mathcal{D}a_n + 2m\bar{A}_m)\eta^m + \frac{1}{2}(m_{mn} + 2mB_{mn})\eta^m\eta^n , \\
\hat{F}_{(4)} &= (\mathcal{D}C + 2B\mathcal{D}A + 2mBB) + (\mathcal{D}T_m + 2B\mathcal{D}a_m + 2A_m\mathcal{D}A + 4mB\bar{A}_m)\eta^m + \\
&\quad + \frac{1}{2}(\mathcal{D}V_{mn} - 4\bar{A}_m\mathcal{D}a_n + 2B_{mn}\mathcal{D}A + 2Bm_{mn} + 4mBB_{mn} - 4m\bar{A}_m\bar{A}_n)\eta^m\eta^n + \\
&\quad + \frac{1}{3!}(\mathcal{D}s_{mnp} + 6B_{mn}\mathcal{D}a_p + 6\bar{A}_m m_{np} + 12m\bar{A}_m B_{np})\eta^m\eta^n\eta^p + \\
&\quad + \frac{1}{4!}(\beta_{mnpq} + 12m_{mn}B_{pq} + 12mB_{mn}B_{pq})\eta^m\eta^n\eta^p\eta^q .
\end{aligned}
\tag{6.4}
$$

As in the case of ordinary type IIA compactified on T^4 [48], the 8 vector fields $(\bar{A}_m, K^m)$ coming from the fields in NS-NS sector transform in the vectorial representation of SO(4,4), 8 scalars (a_m, s_{mnp}) transform in the eight-dimensional spinorial representation R_s, 8 vectors $(A, V_{mn}, \text{dual of } C_{(3)})$ coming from the R-R sector transform in the another spinorial representation R_c, 4 tensor fields T_m split into eight (anti)self-dual 3-form field strengths and there field equations transform covariantly. Finally from the examination of various field strengths we determine that eight masses and fluxes $(m, m_{mn}, \beta_{mnpq})$ fit in the eight dimensional representation R_c. To see this explicitely let us write down the contribution from purely mass terms which follow from the Lorentz scalar contractions of the last terms of the two equations in (6.4),

$$
\begin{aligned}
&\int\left(-\frac{1}{2}\hat{F}_2\ \hat{*}\hat{F}_2-\frac{1}{2}\hat{F}_4\ \hat{*}\hat{F}_4-\frac{m\ \hat{*}m}{2}\right)=\\
&\quad=-\frac{1}{2}\int d^6x\, e\sqrt{G}\Big[(m)^2+\frac{1}{2!}(m_{mn}+2mB_{mn})(m_{pq}+2mB_{pq})G^{mp}G^{nq}+\\
&\qquad\qquad+\frac{1}{4!}(\beta_{mnpq}+12m_{[mn}B_{pq]}+12mB_{[mn}B_{pq]})^2\Big]+\text{other terms}
\end{aligned}
\tag{6.5}
$$

where internal indices are contracted with the metric G_{mn} on the four-torus. Thus the mass m and seven fluxes arrange themselves into a representation, R_c, same as the eight RR vector potentials, and transform under SO(4,4) accordingly. The terms on the right hand side of eq. (6.5) represent SO(4,4) invariant contribution of the mass (or cosmological constant) terms to six-dimensional massive II theory. In other words the massive six-dimensional type-II theory obtained in this way will have SO(4,4) invariance provided the fluxes and masses transform in the 8 dimensional spinorial representation of SO(4,4).

To recall, situation here is analogous to the case of massive type-IIA theory compactified on K_3 [18] where mass and the RR fluxes fit into a vectorial representation of SO(4,20) duality group. However the number of supercharges are double here. The duality group SO(4,4) mixes mass and various fluxes and all of these fluxes on T^4 can be lifted to eleven dimensions, case by case. In the picture below we have summarized a case where 4-form flux on T^4 is lifted to eleven dimensions. The 4-form flux gives rise to 4-form flux in eleven dimensions along the compact four torus. The vacuum solutions with 2-form fluxes would be interesting to study as they will give rise to pure gravity configurations in eleven-dimensions like the domain-wall-instanton in the previous case of T^2.

$$
\begin{array}{ccc}
 & & \boxed{\text{M-theory on } S^1\times T^4\ \&\ \text{4-form-Flux}}\\
 & & \updownarrow\\
\boxed{\text{massive IIA on } T^4} & \xleftrightarrow{(G\leftrightarrow 1/G)} & \boxed{\text{IIA on } T^4\ \&\ \text{4-form-Flux}}
\end{array}
$$

Above sketch is quite similar to a sketch in [18] where M-theory compactification on $S^1\times K3$ with flux is discussed. The M-theory solution for $S^1\times T^4$ compactification with 4-form flux can be obtained by replacing $K3$ line element by a T^4 line element in the eqs. (4.2) of [18].

7. Summary

In this paper we have studied the T^2 compactification of ten-dimensional massive type-IIA theory with Ramond-Ramond background flux corresponding to 2-form field strength. We found that the resulting eight-dimensional theory is a $\mathrm{SL}(2,\mathbb{R})\times\mathrm{SL}(2,\mathbb{R})$ symmetric massive supergravity theory. The mass and the flux parameters transform under $\mathrm{SL}(2,\mathbb{R})$ accordingly. Thus the perturbative T-duality survives even at the massive level when appropriate masses and fluxes are switched on. Next we have shown that this duality symmetry interpolates between vacua of massive type IIA compactified on T^2 and vacua of ordinary type IIA compactified on T^2 with a RR 2-form flux. The wrapped D8-brane solution of massive type IIA turns out to be T-dual to D6-brane solution of ordinary type-IIA theory. This relationship between massless and massive IIA vacuas on T^2 also suggests an 11-dimensional interpretation of massive IIA theory. We find that the D8-brane is related to pure gravity vacua of M-theory which is a direct product of seven-dimensional Minkowski space and 4-dimensional instaton. This Ricci-flat instanton turns out to be a domain-wall generalization of Gibbons-Hawking multi-center Taub-NUT instanton. Thus the compactification of M-theory on such instantons will tell us more deeply about the spectrum of massive type-II sugras in lower dimensions.

We have shown that D6-D8 brane bound states can also exist in massive type IIA similar to the ordinary D$(p-2)$-Dp brane bound states in constant magnetic B-field background. It would be interesting to investigate if there are corresponding non-commutative Yang-Mills theories [46] for D6-D8 bound state.

We also discuss the compactification of massine type IIA on T^4. We find that the mass and the seven R-R fluxes organize themselves into an eight-dimensional representation of duality group SO$(4,4)$. Many of the features of these fluxes will be similar to the case of T^2 compactifications.

Finally let us say something about strong-weak dualities. The full duality symmetry group of type-II theory in eight dimensions is SL$(2,\mathbb{Z})\times$SL$(3,\mathbb{Z})$ which includes the strong-weak duality elements. This non-perturbative duality group cannot be restored in our set-up as we cannot generate more flux parameters than what we already have. However, there is an SL$(3,\mathbb{R})$ symmetric massive type-II action worked out in [49] from T^3 compactification of a massive 11-dimensional sugra [13].

Note added: after this work was communicated I came to know about the overlap between 11-dimensional solution in section 4 and the works in [50, 51].

Acknowledgments

I would like to thank Jan Louis, Thomas Mohaupt and Marco Zagermann for many useful discussions. I am also grateful to M. Zagermann for a very careful reading of the manuscript. This work is supported by AvH (the Alexander von Humboldt foundation).

References

[1] For a review see, O. Aharony, S.S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, *Large-N field theories, string theory and gravity*, *Phys. Rept.* **323** (2000) 183 [hep-th/9905111].

[2] For a collection of reprints see, A. Salam and E. Sezgin eds., *Supergravities in diverse dimensions*, 2 vols., World Scientific 1989.

[3] L.J. Romans, *Massive $N=2a$ supergravity in ten-dimensions*, *Phys. Lett.* **B 169** (1986) 374.

[4] J. Scherk and J.H. Schwarz, *How to get masses from extra dimensions*, *Nucl. Phys.* **B 153** (1979) 61.

[5] E. Bergshoeff, M. de Roo, M.B. Green, G. Papadopoulos and P.K. Townsend, *Duality of type-II 7-branes and 8-branes*, *Nucl. Phys.* **B 470** (1996) 113 [hep-th/9601150].

JHEP04(2002)017

[6] P.M. Cowdall, H. Lü, C.N. Pope, K.S. Stelle and P.K. Townsend, *Domain walls in massive supergravities*, *Nucl. Phys.* **B 486** (1997) 49 [hep-th/9608173].

[7] I.V. Lavrinenko, H. Lü and C.N. Pope, *From topology to generalised dimensional reduction*, *Nucl. Phys.* **B 492** (1997) 278 [hep-th/9611134].

[8] I.V. Lavrinenko, H. Lü and C.N. Pope, *Fibre bundles and generalised dimensional reductions*, *Class. and Quant. Grav.* **15** (1998) 2239 [hep-th/9710243].

[9] H. Lü and C.N. Pope, *Domain walls from M-branes*, *Mod. Phys. Lett.* **A 12** (1997) 1087 [hep-th/9611079].

[10] E. Bergshoeff, M. de Roo and E. Eyras, *Gauged supergravity from dimensional reduction*, *Phys. Lett.* **B 413** (1997) 70 [hep-th/9707130].

[11] E. Bergshoeff, Y. Lozano and T. Ortin, *Massive branes*, *Nucl. Phys.* **B 518** (1998) 363 [hep-th/9712115].

[12] N. Kaloper, R.R. Khuri and R.C. Myers, *On generalized axion reductions*, *Phys. Lett.* **B 428** (1998) 297 [hep-th/9803066].

[13] E. Bergshoeff and J.P. van der Schaar, *On M9-branes*, *Class. and Quant. Grav.* **16** (1999) 23 [hep-th/9806069].

[14] P. Meessen and T. Ortin, *An* $\mathrm{SL}(2,\mathbb{Z})$ *multiplet of nine-dimensional type-II supergravity theories*, *Nucl. Phys.* **B 541** (1999) 195 [hep-th/9806120].

[15] C.M. Hull, *Massive string theories from M-theory and F-theory*, *J. High Energy Phys.* **11** (1998) 027 [hep-th/9811021].

[16] N. Kaloper and R. Myers, *The O(dd) story of massive supergraviy*, *J. High Energy Phys.* **05** (1999) 010 [hep-th/9901045].

[17] M. Cvetič, H. Lü and C.N. Pope, *Gauged six-dimensional supergravity from massive type IIA*, *Phys. Rev. Lett.* **83** (1999) 5226 [hep-th/9906221]; *Brane-world Kaluza-Klein reductions and branes on the brane*, *J. Math. Phys.* **42** (2001) 3048 [hep-th/0009183].

[18] M. Haack, J. Louis and H. Singh, *Massive type IIA theory on K3*, *J. High Energy Phys.* **04** (2001) 040 [hep-th/0102110].

[19] C. Nuñez, I.Y. Park, M. Schvellinger and T.A. Tran, *Supergravity duals of gauge theories from* F_4 *gauged supergravity in six dimensions*, *J. High Energy Phys.* **04** (2001) 025 [hep-th/0103080].

[20] M. Schvellinger and T.A. Tran, *Supergravity duals of gauge field theories from* $\mathrm{SU}(2)\times\mathrm{U}(1)$ *gauged supergravity in five dimensions*, *J. High Energy Phys.* **06** (2001) 025 [hep-th/0105019].

[21] B. Janssen, *Massive T-duality in six dimensions*, *Nucl. Phys.* **B 610** (2001) 280 [hep-th/0105016].

[22] A. Strominger, *Superstrings with torsion*, *Nucl. Phys.* **B 274** (1986) 253.

[23] B. de Wit, D.J. Smit and N.D. Hari Dass, *Residual supersymmetry of compactified* $D=10$ *supergravity*, *Nucl. Phys.* **B 283** (1987) 165.

[24] J. Polchinski and A. Strominger, *New vacua for type-II string theory*, *Phys. Lett.* **B 388** (1996) 736 [hep-th/9510227].

[25] E. Witten, *Strong coupling expansion of Calabi-Yau compactification*, *Nucl. Phys.* **B 471** (1996) 135 [hep-th/9602070].

[26] K. Becker and M. Becker, *M-theory on eight-manifolds*, *Nucl. Phys.* **B 477** (1996) 155 [hep-th/9605053].

[27] J. Michelson, *Compactifications of type-IIB strings to four dimensions with non-trivial classical potential*, *Nucl. Phys.* **B 495** (1997) 127 [hep-th/9610151].

[28] A. Lukas, B.A. Ovrut and D. Waldram, *Stabilizing dilaton and moduli vacua in string and M-theory cosmology*, *Nucl. Phys.* **B 509** (1998) 169 [hep-th/9611204].

[29] H.P. Nilles, M. Olechowski and M. Yamaguchi, *Supersymmetry breaking and soft terms in M-theory*, *Phys. Lett.* **B 415** (1997) 24 [hep-th/9707143].

[30] A. Lukas, B.A. Ovrut and D. Waldram, *On the four-dimensional effective action of strongly coupled heterotic string theory*, *Nucl. Phys.* **B 532** (1998) 43 [hep-th/9710208].

[31] A. Lukas, B.A. Ovrut, K.S. Stelle and D. Waldram, *Heterotic M-theory in five dimensions*, *Nucl. Phys.* **B 552** (1999) 246 [hep-th/9806051].

[32] S. Gukov, C. Vafa and E. Witten, *CFT's from Calabi-Yau four-folds*, *Nucl. Phys.* **B 584** (2000) 69 [hep-th/9906070].

[33] K. Dasgupta, G. Rajesh and S. Sethi, *M-theory, orientifolds and g-flux*, *J. High Energy Phys.* **08** (1999) 023 [hep-th/9908088].

[34] S. Gukov, *Solitons, superpotentials and calibrations*, *Nucl. Phys.* **B 574** (2000) 169 [hep-th/9911011].

[35] T.R. Taylor and C. Vafa, *RR flux on Calabi-Yau and partial supersymmetry breaking*, *Phys. Lett.* **B 474** (2000) 130 [hep-th/9912152].

[36] K. Behrndt and S. Gukov, *Domain walls and superpotentials from m theory on Calabi-Yau three-folds*, *Nucl. Phys.* **B 580** (2000) 225 [hep-th/0001082].

[37] P. Mayr, *On supersymmetry breaking in string theory and its realization in brane worlds*, *Nucl. Phys.* **B 593** (2001) 99 [hep-th/0003198].

[38] M. Spalinski and T.R. Taylor, *Branes and fluxes in $D = 5$ Calabi-Yau compactifications of M-theory*, *Phys. Lett.* **B 485** (2000) 263 [hep-th/0004095].

[39] B.R. Greene, K. Schalm and G. Shiu, *Warped compactifications in M- and F-theory*, *Nucl. Phys.* **B 584** (2000) 480 [hep-th/0004103].

[40] M. Haack, J. Louis and M. Marquart, *Type-IIA and heterotic string vacua in $D = 2$*, hep-th/0011075.

[41] G. Curio and A. Krause, *Four-flux and warped heterotic M-theory compactifications*, *Nucl. Phys.* **B 602** (2001) 172 [hep-th/0012152].

[42] G. Curio, A. Klemm, D. Lüst and S. Theisen, *On the vacuum structure of type-II string compactifications on Calabi-Yau spaces with H-fluxes*, *Nucl. Phys.* **B 609** (2001) 3 [hep-th/0012213];
G. Curio, A. Klemm, B. Kors and D. Lüst, *Fluxes in heterotic and type-II string compactifications*, *Nucl. Phys.* **B 620** (2002) 237 [hep-th/0106155].

[43] J. Maharana and J.H. Schwarz, *Noncompact symmetries in string theory*, *Nucl. Phys.* **B 390** (1993) 3 [hep-th/9207016].

[44] G.W. Gibbons and S.W. Hawking, *Gravitational multi-instantons*, *Phys. Lett.* **B 78** (1978) 430.

[45] J.X. Lu and S. Roy, *(p+1)-dimensional noncommutative Yang-Mills and D$(p-2)$ branes*, *Nucl. Phys.* **B 579** (2000) 229 [hep-th/9912165].

[46] N. Seiberg and E. Witten, *String theory and noncommutative geometry*, *J. High Energy Phys.* **09** (1999) 032 [hep-th/9908142].

[47] H. Singh, work in progress.

[48] A. Sen and C. Vafa, *Dual pairs of type-II string compactification*, *Nucl. Phys.* **B 455** (1995) 165 [hep-th/9508064].

[49] N. Alonso-Alberca, P. Meessen and T. Ortin, *An* $\mathrm{SL}(3,\mathbb{Z})$ *multiplet of 8-dimensional type-II supergravity theories and the gauged supergravity inside*, *Nucl. Phys.* **B 602** (2001) 329 [hep-th/0012032].

[50] G.W. Gibbons and P. Rychenkova, *Single-sided domain walls in M-theory*, *J. Geom. Phys.* **32** (2000) 311 [hep-th/9811045].

[51] G.W. Gibbons, H. Lu, C.N. Pope and K.S. Stelle, *Supersymmetric domain walls from metrics of special holonomy*, *Nucl. Phys.* **B 623** (2002) 3 [hep-th/0108191].